Grade Level/Course: Grade 6
Lesson/Unit Plan Name: Factoring Expressions and Variable Expressions
Rationale/Lesson Abstract: Students will learn to compare common factors of whole number and variable expressions and re-write equivalent expressions using the distributive property.
Timeframe: 60-90 minutes
Common Core Standard(s): 6.NS.4: Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).* 6.EE.3: Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

Instructional Resources/Materials:
1) 2-sided counters (30 counters for every 2 students)
2) Copies of Warm-Up (If passing out. Warm-Up may also be shown on a document camera.)

Warm-Up Answer Key (Master copies on page 7):

<u>Key to Warm-Up:</u>

1) **Yes**: The sum of the products 60 and 24.

2) No: $10 \bullet 10$ or $10(6+4)$

3) **Yes:** Repetitive addition of the quantity 10+4 (14).

4) No: $14 \bullet 5$

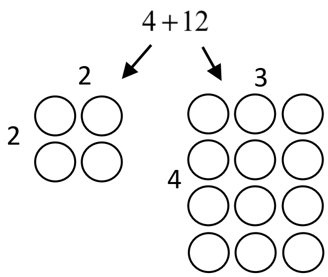
5) **Yes:** Generic rectangle showing partial products 60 and 24 representing a product (area) of 84.

6) **Yes:** Bar model showing repetitive addition of the quantity 14.

Activity/Lesson:

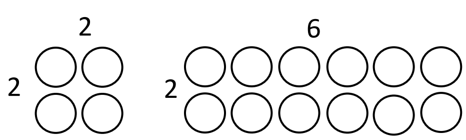
Example 1 (I Do): Use the distributive property to factor out the GCF and write an expression equivalent to $4+12$.

2-Sided Counters:

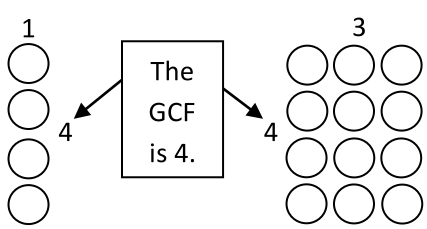


*Create 2 arrays that represent 4 and 12. The area represents the value and the two dimensions are the factors of the value.

*Build the arrays so that they **share the largest common dimension possible**.



NOTE: 2 is a common factor, but not the greatest common factor. Allow students to build the arrays until they discover the largest common dimension possible.



$$\therefore 4+12=4(1)+4(3)$$
$$=4(1+3)$$ ← Equivalent expression

Generic Rectangle:

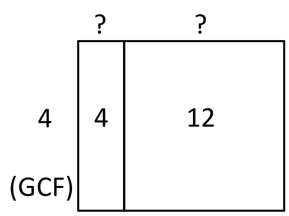
*Find the GCF of 4 and 12. This will be the common dimension of the rectangle.

$4=2\bullet 2$

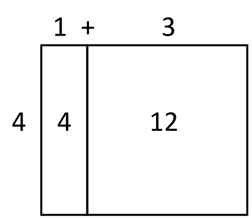
$12=2\bullet 2\bullet 3$

$\text{GCF}=2\bullet 2$

$\text{GCF}=4$



*The partial areas represent the products of 4 and two other factors. The other factors must be 1 and 3.



$$\therefore 4+12=4(1+3)$$

Decomposition:

*Decompose 4 and 12 to prime factors.

$4+12$

$=(2\bullet 2)+(2\bullet 2\bullet 3)$

*Use the associative property to group all common prime factors of 4 and 12. Note: 1 is a factor of every number.

$=(2\bullet 2)(1)+(2\bullet 2)(3)$

$=4(1)+4(3)$

*The product of <u>all common prime factors</u> is the GCF.

$=4(1+3)$

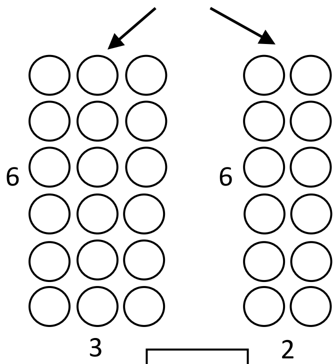
*Use the distributive property to re-write an equivalent expression.

Activity/Lesson continued:

Example 2: (Partner Do)

2-Sided Counters:

$18+12$



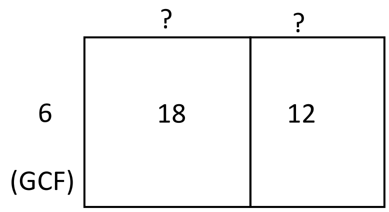
*Allow students to create as many arrays as needed to find the largest common dimension.

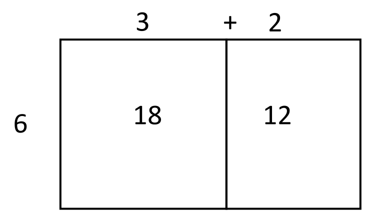
The GCF is 6.

$$\therefore 18+12=6(3)+6(2)$$
$$=6(3+2)$$

Generic Rectangle:

* The GCF of 18 and 12 is 6.





$$\therefore 18+12=6(3+2)$$

Decomposition:

*Decompose 18 and 12 to prime factors.

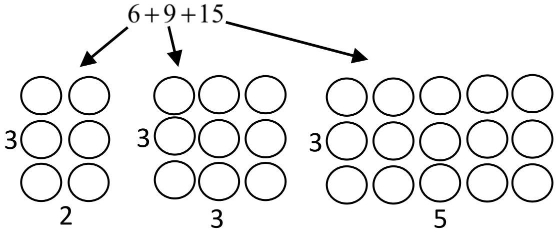
$$18+12$$
$$=(2\bullet 3\bullet 3)+(2\bullet 2\bullet 3)$$
$$=(2\bullet 3)(3)+(2\bullet 3)(2)$$
$$=6(3)+6(2)$$
$$=6(3+2)$$

*Use commutative and associative properties to re-group all common prime factors.

*Use distributive property to re-write an equivalent expression.

Example 3: (Partner Do)

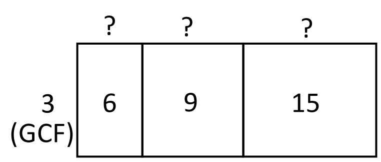
$6+9+15$

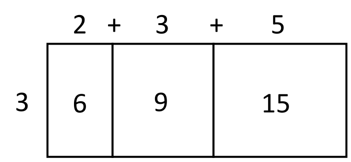


The GCF is 3.

$$\therefore 6+9+15=3(2)+3(3)+3(5)$$
$$=3(2+3+5)$$

* The GCF of 6, 9, and 15 is 3.



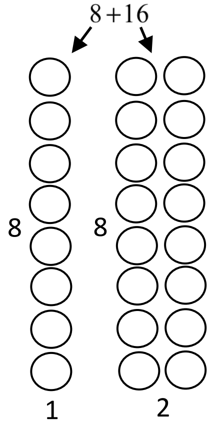


$$\therefore 6+9+15=3(2+3+5)$$

$$6+9+15$$
$$=(2\bullet 3)+(3\bullet 3)+(3\bullet 5)$$
$$=(3)(2)+(3)(3)+(3)(5)$$
$$=3(2)+3(3)+3(5)$$
$$=3(2+3+5)$$

Example 4: (You Do)

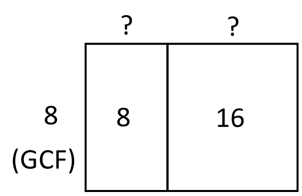
$8+16$

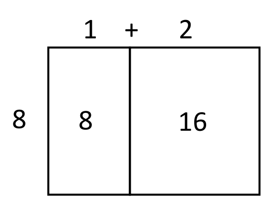


*Note: Students may have difficulty including 1 as a factor when re-writing an equivalent expression. Remind students that 1 is a factor of every number (identity property of multiplication).

$$\therefore 8+16=8(1)+8(2)$$
$$=8(1+2)$$

The GCF of 8 and 16 is 8.





$$8+16$$
$$=(2\bullet 2\bullet 2)+(2\bullet 2\bullet 2\bullet 2)$$
$$=(2\bullet 2\bullet 2)(1)+(2\bullet 2\bullet 2)(2)$$
$$=8(1)+8(2)$$
$$=8(1+2)$$

Activity/Lesson continued:

Example 5: (We Do)

2-Sided Counters:

$105 + 120$

*Point out to students that this approach is not an efficient method for this problem due to the values of each term.

Generic Rectangle:

*Remind students to decompose each term to prime factors to find the GCF.

$120 = 2 \bullet 2 \bullet 2 \bullet 3 \bullet 5$

$105 = 3 \bullet 5 \bullet 7$

$GCF = 3 \bullet 5$

$GCF = 15$

	8	+ 7
15	120	105

$\therefore 120 + 105 = 15(8 + 7)$

Decomposition:

*Note: Students have already decomposed each term into a product of its primes. Have students demonstrate proper syntax when re-writing the expression.

$120 + 105$

$= (2 \bullet 2 \bullet 2 \bullet 3 \bullet 5) + (3 \bullet 5 \bullet 7)$

$= (3 \bullet 5)(2 \bullet 2 \bullet 2) + (3 \bullet 5)(7)$

$= 15(8) + 15(7)$

$= 15(8 + 7)$

Example 6 (You Do)

Generic Rectangle:

$48 + 120$

$120 = 2 \bullet 2 \bullet 2 \bullet 3 \bullet 5$

$48 = 2 \bullet 2 \bullet 2 \bullet 3 \bullet 2$

$GCF = 2 \bullet 2 \bullet 2 \bullet 3$

$GCF = 24$

	2	+ 5
24	48	120

$\therefore 48 + 120 = 24(2 + 5)$

Decomposition:

$48 + 120$

$= (2 \bullet 2 \bullet 2 \bullet 2 \bullet 3) + (2 \bullet 2 \bullet 2 \bullet 3 \bullet 5)$

$= (2 \bullet 2 \bullet 2 \bullet 3)(2) + (2 \bullet 2 \bullet 2 \bullet 3)(5)$

$= 24(2) + 24(5)$

$= 24(2 + 5)$

Activity/Lesson continued:

Example 7: Connection to Algebra (We Do)

Generic Rectangle:

$$6y + 42y^2$$

$6y = 2 \bullet 3 \quad \bullet y$

$42y^2 = 2 \bullet 3 \bullet 7 \bullet y \bullet y$

$\text{GCF} = 2 \bullet 3 \bullet \quad y$

$\text{GCF} = 6y$

$1 \quad + \quad 7y$

$6y$	$6y$	$42y^2$

$\therefore 6y + 42y^2 = 6y(1 + 7y)$

Decomposition:

$$6y + 42y^2$$

$= (2 \bullet 3 \bullet y) + (2 \bullet 3 \bullet 7 \bullet y \bullet y)$

$= (2 \bullet 3 \bullet y)(1) + (2 \bullet 3 \bullet y)(7 \bullet y)$

$= 6y(1) + 6y(7y)$

$= 6y(1 + 7y)$

Example 8: (You Do)

$$2xy + 10x^2$$

$2xy = 2 \bullet y \bullet x$

$10x^2 = 2 \bullet 5 \bullet x \bullet x$

$\text{GCF} = 2 \bullet \quad x$

$\text{GCF} = 2x$

$y \quad + \quad 5x$

$2x$	$2xy$	$10x^2$

$\therefore 2xy + 10x^2 = 2x(y + 5x)$

$$2xy + 10x^2$$

$= (2 \bullet x \bullet y) + (2 \bullet 5 \bullet x \bullet x)$

$= (2 \bullet x)(y) + (2 \bullet x)(5 \bullet x)$

$= 2x(y) + 2x(5x)$

$= 2x(y + 5x)$

Assessment:

Ticket out the Door Problems: Give students scratch paper and write both problems on the board. Check for student understanding before students leave the room.

Use the distributive property to write an equivalent expression. Do each problem 2 ways.

1) Generic Rectangle:

$18 + 114$

$114 = 2 \bullet 3 \bullet 19$

$18 = 2 \bullet 3 \bullet 3$

$\text{GCF} = 2 \bullet 3$

$\text{GCF} = 6$

	3 +	19
6	18	114

$\therefore 18 + 114 = 6(3 + 19)$

Decomposition:

$18 + 114$

$= (2 \bullet 3 \bullet 3) + (2 \bullet 3 \bullet 19)$

$= (2 \bullet 3)(3) + (2 \bullet 3)(19)$

$= 6(3) + 6(19)$

$= 6(3 + 19)$

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2) Generic Rectangle:

$12y^2 + 16x$

$12y^2 = 2 \bullet 2 \bullet 3 \bullet y \bullet y$

$16x = 2 \bullet 2 \bullet 2 \bullet 2 \bullet x$

$\text{GCF} = 2 \bullet 2$

$\text{GCF} = 4$

	$3y^2$ +	$4x$
4	$12y^2$	$16x$

$\therefore 12y^2 + 16x = 4(3y^2 + 4x)$

Decomposition:

$12y^2 + 16x$

$= (2 \bullet 2 \bullet 3 \bullet y \bullet y) + (2 \bullet 2 \bullet 2 \bullet 2 \bullet x)$

$= (2 \bullet 2)(3 \bullet y \bullet y) + (2 \bullet 2)(2 \bullet 2 \bullet x)$

$= 4(3y^2) + 4(4x)$

$= 4(3y^2 + 4x)$

Warm-Up:

Select "Yes" for all choices that represent the multiplication problem $14 \bullet 6$ and "No" for choices that do not. For all "No" choices, write the correct multiplication problem.

1) $(6 \bullet 10)+(6 \bullet 4)$ — Yes No

2)

	6	+ 4
10	60	40

Yes No

3) $(10+4)+(10+4)+(10+4)+(10+4)+(10+4)+(10+4)$ — Yes No

4) $14+14+14+14+14$ — Yes No

5)

	10	+ 4
6	60	24

Yes No

6)

14	14	14	14	14	14

Yes No

Select "Yes" for all choices that represent the multiplication problem $14 \bullet 6$ and "No" for choices that do not. For all "No" choices, write the correct multiplication problem.

1) $(6 \bullet 10)+(6 \bullet 4)$ — Yes No

2)

	6	+ 4
10	60	40

Yes No

3) $(10+4)+(10+4)+(10+4)+(10+4)+(10+4)+(10+4)$ — Yes No

4) $14+14+14+14+14$ — Yes No

5)

	10	+ 4
6	60	24

Yes No

6)

14	14	14	14	14	14

Yes No